BANK PROBATIONARY
REASONING ABILITY

PROBLEMS ON CUBES & CUBOIDS

Cube is a solid body which has 6 faces, 12 edges (AB, BC, CD, AD, AE, BF, DH, CG, EF, FG, GH and EH) and 8 corners (A, B, C, D, E, F, G and H). Each face of the cube is square in shape and all faces are congruent squares. Hence, if the edge length of the cube is ‘a’ units, each edge has the length ‘a’ units.

Volume of a cube of edge length ‘a’ units
= a³ cubic units.

By the term ‘unit cube’, we mean a cube with edge length 1 unit
∴ Volume of a unit cube = (1³ = 1) 1 cubic unit

Volume of a cube of edge length ‘a’ units
= sum of the volumes of the unit cubes used to form the given cube
= 1 + 1+ .......(a³ times)
= a³ cubic units

Hence, if a cube of edge length ‘a’ units is divided into unit cubes the number of unit cubes will be equal to the volume of the cube, i.e., a³

For example, if a cube of edge length 4 cm is divided into unit cubes, then the number of unit cubes will be (4)³ = 64. If a cube of edge length 6 cm is divided into unit cubes, the number of unit cubes will be (6)³ = 216

In general, a cube of edge length ‘a’ units can be divided into ‘a³’ unit cubes i.e. the number is equal to the volume of the cube

This idea is used for solving problems on cubes

Now, out of the a³ of there a³ unit cubes, there are 4 different types of cubes: 4 different types of cubes:

i) Cubes with three face visible
ii) Cubes with two face visible
iii) Cubes with one face visible
iv) Cubes with no face visible

The cubes with three faces visible are the cubes at the corners. Hence, the number of cubes whose three faces are visible is equal to the number of corners, i.e. 8

No of cubes whose three faces are visible = 8

The cubes with two faces visible are the cubes at the edges. But, if the edge length is ‘a’ units, the number of unit cubes in an edge is ‘a’. But, we need only the cubes with two faces visible. Therefore, out of the ‘a’ cubes in the edge, we have to remove the one’s whose three faces are visible (i.e. the cubes at the corners which is 2). Hence, the remaining number i.e. (a-2) will be the number of cubes in one edge whose two faces are visible. There is a total of 12 edges. Therefore, the total number of cubes with two faces visible is 12(a - 2)

∴ No of cubes with two faces visible
= 12 (a - 2) similarly

No. of cubes with one face visible = 6(a - 2)²
No. of cubes with no face visible = (a - 2)³
A cube of edge length 'a' units can be divided into 'a^3' unit cubes.

- No. of unit cubes with three face visible = 8
- No. of unit cubes with two face visible = 12 (a - 2)
- No. of unit cubes with one face visible = 6 (a - 2)^2
- No. of unit cubes with no face visible = (a - 2)^3

Same is the case for Cuboid

A Cuboid is a solid body with each face being a rectangle. Hence, Cuboid has length, breadth and height.

Let the length, breadth and thickness (height) be a, b and c respectively

Volume of Cuboid

= length x breadth x height
= abc cubic units

Here also, the number of unit cubes into which this cuboid can be divided is equal to the volume of the cuboid (i.e., a x b x c)

A cuboid having length a, breadth b and height c is divided into unit cubes, i.e., a x b x c unit cubes (= volume)

- No. of unit cubes whose three face are visible = 8
- No. of unit cubes whose two faces are visible
  = 4(a - 2) + 4(b - 2) + 4(c - 2) = 4 [a + b + c - 6]
- No. of unit cubes whose one face is visible
  = 2 [(a - 2)(b - 2) + (b - 2)(c - 2) + (a - 2)(c - 2)]
- No. of unit cubes whose no face is visible
  = (a - 2)(b - 2)(c - 2)

Note that, for solving problems on cubes and cuboids, we make use of the concept of unit cubes and the formulas presented in this chapter

Solved Examples:

Direction (Q.1-5) : A cube is painted red and all its faces and is then divided into 343 equal cubes.

1. How many cubes will have three faces painted?
   1) 0  2) 8  3) 16  4) 4  5) None of these
2. How may cubes will have two faces painted?
   1) 55  2) 70  3) 45  4) 50  5) None of these
3. How many cubes will have one face painted?
   1) 112  2) 148  3) 162  4) 150  5) None of these
4. How many cubes have no face painted?
   1) 216  2) 64  3) 125  4) 27  5) None of these
5. How many cubes will have at least one face painted?
   1) 178  2) 218  3) 127  4) 279  5) None of these

Solution:

Since the cube is divided into 343 equal cubes i.e. (7)^3 equal cubes, assume the edge length of the cube to be 7, (a = 7).

This is done using the idea of unit cubes. (A cube of edge length a unit can be divided into a^3 unit cubes)

1. (2) The number of cubes whose three face are painted is equal to the number of cubes whose three faces are visible. (Since only those faces which are visible can be painted). Hence, the number is 8.
2. (5) The number of cubes whose two faces are painted
   = The number of cubes whose two faces are visible
   = 12 (a - 2) = 12 (7 - 2) = 60
3. (4) The number of cubes whose one face is painted
   = The number of cubes who one face is visible
\[ a = 6(a - 2)^2 = 6(7 - 2)^2 \]
\[ a = 150 \]

4.(3); The number of cubes where no face is painted
\[ = (a - 2)^3 = (7 - 2)^3 \]
\[ = 125. \]

5. (2) The number of cubes whose at least one face is painted
\[ = (b - 2) \times \text{ face painted} + (c - 2) \times \text{ face painted} \]
\[ = \text{ Total} - \text{ (No face visible)} \]
\[ = 343 - 125 \]
\[ = 218 \]

Directions (Q.6-10): Read the following informations carefully and answer the questions that follow:

i) There is a rectangular wooden block of length 8cm, height 6cm and breadth 5cm.

ii) The two opposite surfaces of 8cm x 6cm are painted from outside by yellow colour.

iii) One surface of 8cm x 5cm is painted from outside by red colour and the opposite surface brown.

iv) The remaining two surfaces of 6cm x 5cm are painted outside by orange and black respectively.

v) Now, the block is cut in such a way that cubes of 1cm x 1cm x 1cm are created?

6. How many cubes have only one side painted?
   1) 82   2) 122   3) 114   4) 96   5) None of these

7. How many cubes have no side painted?
   1) 62   2) 70   3) 74   4) 82   5) None of these

8. How many cubes have combination of yellow and red only?
   1) 8   2) 12   3) 16   4) 20   5) None of these

9. How many cubes have the combination of brown with yellow and black?
   1) nil   2) one   3) two   4) three   5) None of these

10. How many cubes have two sides painted and remaining sides unpainted?
    1) 46   2) 48   3) 50   4) 52   5) None of these

Solution:

Whenever the block is cut in cubes of 1cm x 1cm x 1cm.

The following are the formulas for finding the:

i) No surface coloured=(a-2) (b-2) (c-2)

ii) One surface coloured
\[ = 2(a-2)(b-2)+2(b-2) (c-2)+2(a-2) (c-2) \]

iii) Two surfaces coloured=4[a+b + c - 6]

iv) Three surface coloured = 8

6. (5); \[ 2(a - 2)(b - 2) + 2(b - 2)(c - 2) + 2(a - 2)(c - 2) \]
\[ = 2 \times 6 \times 3 + 2 \times 3 \times 4 + 2 \times 6 \times 4 \]
\[ = 36 + 24 + 48 = 108 \]

7.(5); \[ (a - 2)(b - 2)(c - 2) \]
\[ = 6 \times 3 \times 4 \]
\[ = 72 \]
8. (2):
Cubes which have combination of yellow and red only = 2(a - 2) = 2 x (8 - 2) = 12

[Since only two edges are to be considered and each edge is of length a cm, removing the cubes in the corners of each edge, (two surface painted), we have (a - 2) in one edge. Therefore, from two edges, we have 2(a - a)]

9. (2)
10.(4):
\[4[a + b + c - 6] = 4[8 + 5 + 6 - 9] = 52\]

**Exercises:**

**Directions (Qs. 1 - 4):** Read the following information and answer questions given below it.

i) There is a rectangular wooden block of length 4cm, height 3cm and breadth 3cm.
ii) The two opposite surfaces of 4cm x 3cm are painted from outside by yellow colour.
iii) The other two opposite surfaces of 4cm x 3cm are painted from outside by red colour.
iv) The remaining two surfaces of 3cm x 3cm are painted from outside by green colour.
v) Now, the block is cut in such a way that cubes of 1cm x 1cm x 1cm are created.

1. How many cubes will have only one colour?
   1) 10 2) 12 3) 14 4) 18 5) None of these

2. How many cubes will have only one colour?
   1) 1 2) 2 3) 4 4) 8 5) None of these

3. How many cubes will have all the three colours: red, yellow and green?
   1) 32 2) 24 3) 16 4) 12 5) None of these

4. How many cubes will have all the three colours?
   1) 5 2) 10 3) 8 4) 16 5) None of these

**Directions (Qs. 5 - 10):** Read the following informations and answer questions given below it:

i) There is a wooden block which is cubical in shape and having edge length 6cm.
ii) The opposite surfaces are painted from outside by red, yellow and orange respectively.
iii) Now, the block is cut in such a way that cubes of 1cm x 1cm x 1cm are created.

5. How many cubes will have any two colors?
   1) 48 2) 36 3) 44 4) 40 5) None of these

6. How many cubes will have only orange color?
   1) 16 2) 32 3) 96 4) 48 5) None of these

7. How many cubes will have at least one colour?
   1) 216 2) 152 3) 64 4) 96 5) None of these

8. How many cubes will have only two colours yellow and orange?
   1) 64 2) 125 3) 8 4) 48 5) None of these

9. How many cube will have at least three sides painted
   1) 8 2) 16 3) 6 4) 12 5) None of these

10. If instead the block into cubes of 1cm x 1cm x 1cm, if the block is cut in such a way that cubes of 2cm x 2cm x 2cm are created, how many cubes will be created?
    1) 9 2) 64 3) 36 4) 27 5) None of these

**Answers**

1) (1) 2) (2) 3) (5) 4) (3) 5) (1) 6) (2) 7) (2) 8) (5) 9) (1) 10) (4)